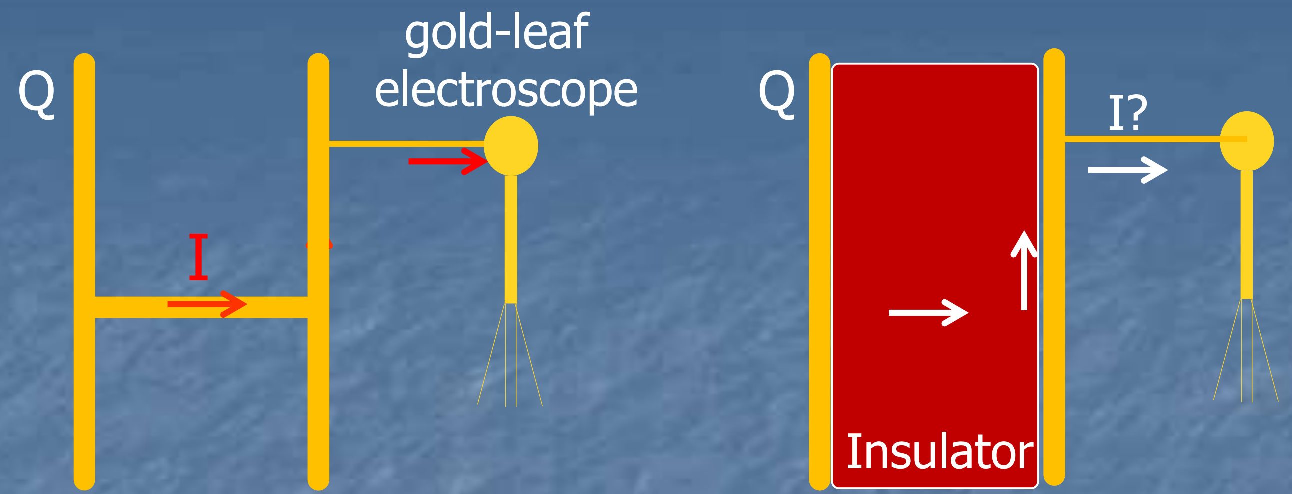
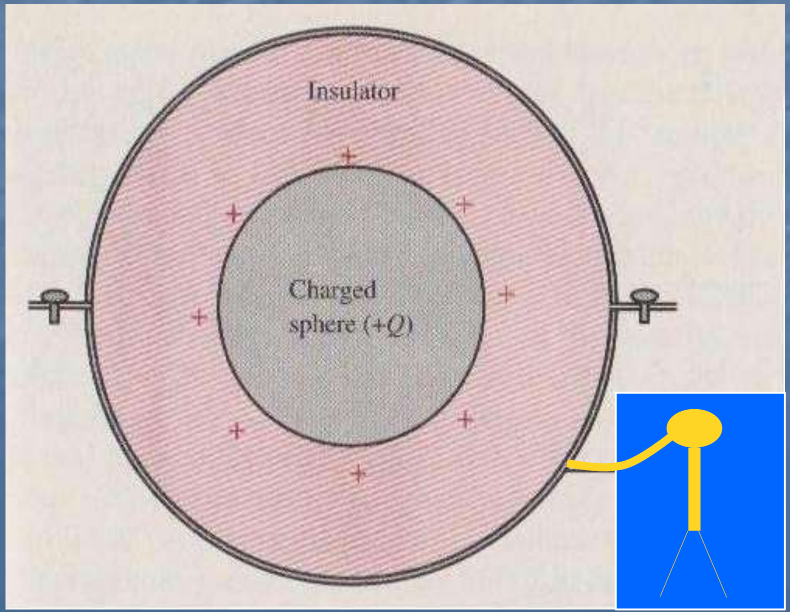
Examples

History of the displacement vector D



Michael Faraday
(1791 –1867)

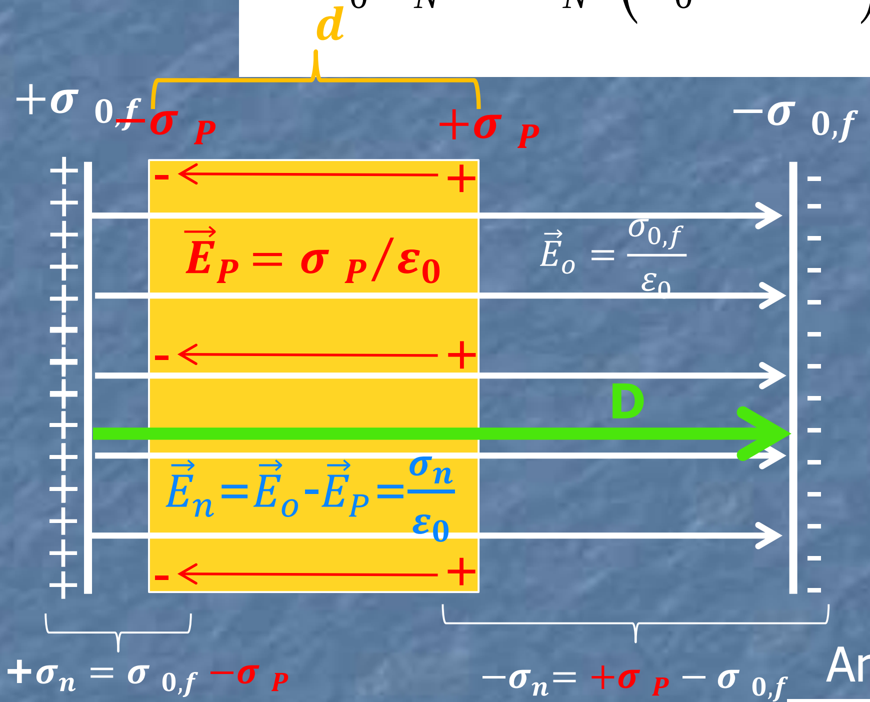
- Faraday tried many types of insulators but never prevented the displacement of charge
- The charge displaced was always Q, equal to that placed on the left plate.
- A sketch of the original system used by Faraday:



The relation between $\epsilon_0, \epsilon, \epsilon_r \vec{D}, \vec{E}$ in Linear-Isotropic-Homogeneous medium

$$\oiint_S \vec{E}\cdot d\vec{s} = \oiint_S E_N ds = \frac{q_0}{\varepsilon_0}\Bigg| \times \varepsilon_0 \Rightarrow \oiint_S \varepsilon_0 E_N ds = q_0 \rightarrow$$

$$\rightarrow \varepsilon_0 E_N \equiv D_N \;\left(\varepsilon_0 \vec{E} = \vec{D}\right) \Rightarrow \oiint_S D_N ds = q_0 = \oiint_S \overset{\{Cm^{-2}\}}{\vec{D}} \cdot d\vec{s} \Rightarrow div\vec{D} = \rho_0$$



The polarization per unit volume:

$$P = \frac{p}{V} = \frac{Q_P \cdot d}{V} = \frac{\sigma_P \cdot A \cdot d}{V} = \frac{\sigma_P \cdot V}{V} = \sigma_P \qquad [1]$$

The relation between $\vec{E}$ and $\vec{P}$ is:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad [2]$$

$$\vec{D} = \varepsilon_0 \vec{E}_0 = \varepsilon_1 \vec{E}_1 = \varepsilon_i \vec{E}_i \;\; ; \;\; \left\{\frac{q_o}{S}\right\} = \{\sigma_o\} \qquad [3]$$

And therefore the resultant field in the medium is:

$$E_N = E = E_0 - E_P = \left(E_0 - \frac{\sigma_P}{\varepsilon_0}\right) / \cdot \varepsilon_0 \Rightarrow$$

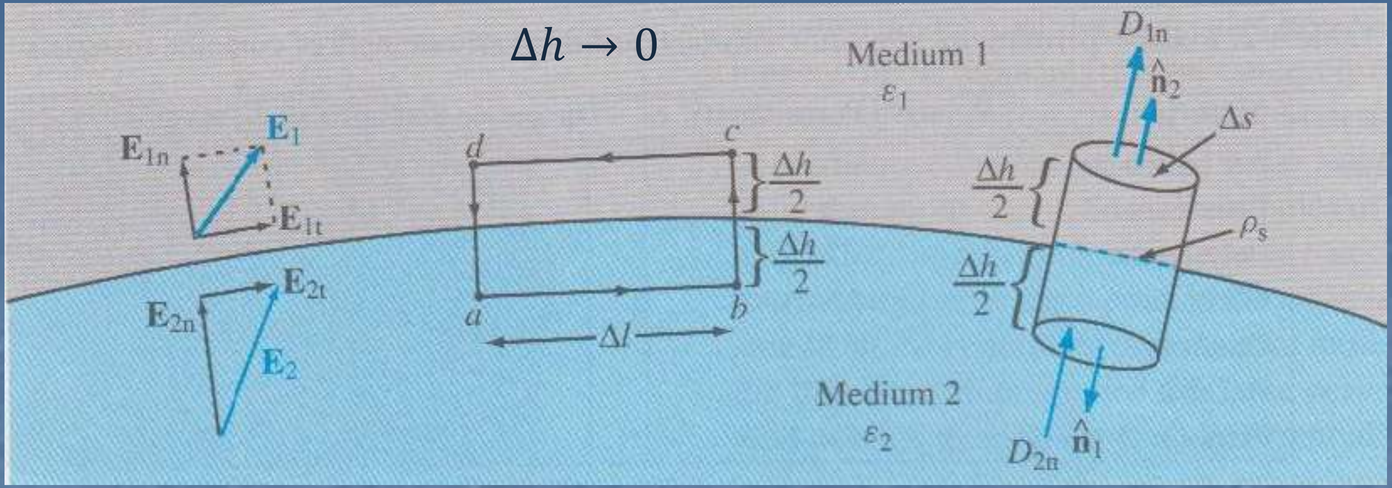
$$\Rightarrow \varepsilon_0 E = \varepsilon_0 E_0 - \sigma_P = \varepsilon_0 E_0 - P = D - P = D - \varepsilon_0 \chi_e E \Rightarrow$$

$$\Rightarrow D = \varepsilon_0 E + \varepsilon_0 \chi_e E = \varepsilon_0 (1 + \chi_e) E \equiv \varepsilon E \Rightarrow (1 + \chi_e) = \frac{\varepsilon}{\varepsilon_0} \equiv \varepsilon_r > 1$$

unitless

A privet case of orthogonal fields:

Boundary condition for the electric field



Is this is true in the case of time dependent fields?

$$1.\quad 0=\oint_c \vec{E}\cdot d\vec{l} = \int_a^b E_{1T}dl + \int_c^d -E_{2T}dl \Rightarrow E_{1T} = E_{2T} \Rightarrow \frac{D_{1T}}{\varepsilon_1} = \frac{D_{2T}}{\varepsilon_2}$$

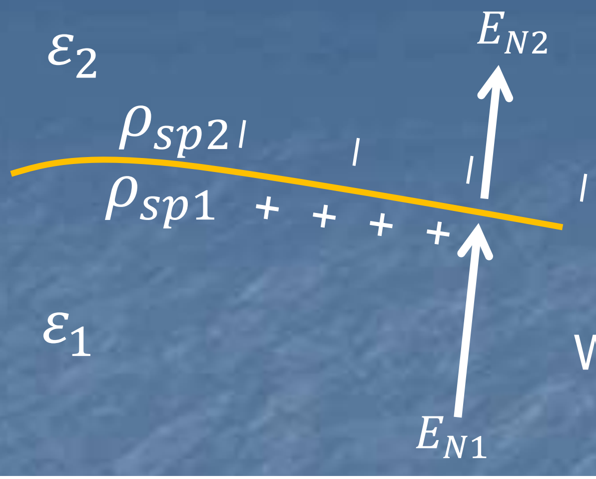
$$2.\quad Q_s = \oint_s \vec{D}\cdot d\vec{s} = \int_{top} D_{1N}ds + \int_{bottom} -D_{2N}ds = \left(D_{1N} - D_{2N}\right)\Delta s \Rightarrow \frac{Q_s}{\Delta s} = \rho_s = D_{1N} - D_{2N} \Rightarrow$$

$$\Rightarrow \varepsilon_1 E_{1N} - \varepsilon_2 E_{2N} = \rho_s$$

$$When\ \rho_s = 0 \Rightarrow (3)\ D_{1N} = D_{2N} \quad and \quad (4)\ \varepsilon_1 E_{1N} = \varepsilon_2 E_{2N}$$

An important emphasis: ρ_s *is not* ρ_{sp}

Regarding the charge on the interface plan:



$\rho_{spn} = \rho_{sp1} - \rho_{sp2}$

Why is ρ_{spn} not expressed in Equation 4?

$$When\ \rho_s = 0\ ,\quad (4)\ \boxed{\varepsilon_1 E_{N1} = \varepsilon_2 E_{N2}}\ ;\ \text{int}\,roducing\quad \varepsilon = \varepsilon_0 \varepsilon_r :$$

$$\varepsilon_0 \varepsilon_{r1} E_{N1} = \varepsilon_0 \varepsilon_{r2} E_{N2}\ ;\quad \text{int}\,roducing\quad \varepsilon_r = 1 + \chi :$$

$$\varepsilon_0 \left(1 + \chi_1\right) E_{N1} = \varepsilon_0 \left(1 + \chi_2\right) E_{N2} \Rightarrow$$

$$\varepsilon_0 E_{N1} + \varepsilon_0 \chi_1 E_{N1} = \varepsilon_0 E_{N2} + \varepsilon_0 \chi_2 E_{N2}\ ;\ recalling\ \ that\ P = \varepsilon_0 \chi E :$$

$$\varepsilon_0 E_{N1} + P_{N1} = \varepsilon_0 E_{N2} + P_{N2}\ ;\ recalling\ \ that\ P_N = \rho_{sp} :$$

$$\Rightarrow \varepsilon_0 \left(E_{N1} - E_{N2}\right) = P_{N2} - P_{N1} = \rho_{sp2} - \rho_{sp1}\quad and\ \ therefore :$$

$$\boxed{E_{N1} - E_{N2} = \frac{\rho_{sp2} - \rho_{sp1}}{\varepsilon_0} = \frac{\rho_{spn}}{\varepsilon_0}}\quad (5)$$

As in a capacitor

The advantage of Eq. 4 over 5 is due to the value of ε being known, while ρ_{spn} is unknown and to be calculated.

The particular case of E_N

The electric fields of point charge, infinite wire, charged sphere, infinite charged plane and more, where all calculated for vector field perpendicular to the medium surface. Assuming medium-vaccume interface, one get from Eq. 4:

$$(4)\ \varepsilon_1 E_{1N} = \varepsilon_2 E_{2N} \Rightarrow \varepsilon_0 E_0 = \varepsilon E \Rightarrow E = \frac{\varepsilon_0}{\varepsilon} E_0 = \frac{E_0}{\varepsilon_r}; \qquad \boxed{E = \frac{E_0}{\varepsilon_r}}$$

That is to say that the field in vacuum is ε_r times greater than that in medium. Employed this on the case of electric field of a point charge one gets:

$$E = \frac{E_0}{\varepsilon_r} = \frac{\dfrac{q}{4\pi\varepsilon_0 r^2}}{\varepsilon_r} = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{q}{4\pi\varepsilon r^2}$$

and for homogeneously charged infinite plane:

$$E = \frac{E_0}{\varepsilon_r} = \frac{\dfrac{\rho_s}{\varepsilon_0}}{\varepsilon_r} = \frac{\rho_s}{\varepsilon_r\varepsilon_0} = \frac{\rho_s}{\varepsilon}$$

and so on… and therefore in all the expressions of electric field one should introduces ε instead of ε_0.

Is crossing the interface between two media might change the direction of the electric field vector and how?

As for the electric potential:

$$\frac{V}{V_0} = \frac{-\int_a^b \vec{E}\cdot d\vec{l}}{-\int_a^b \vec{E}_0\cdot d\vec{l}} = \frac{-\int_a^b \dfrac{\vec{E}_0}{\varepsilon_r}\cdot d\vec{l}}{-\int_a^b \vec{E}_0\cdot d\vec{l}} = \frac{1}{\varepsilon_r} \Rightarrow \qquad V = \frac{V_0}{\varepsilon_r}$$

As for the electric flux:

$$\frac{\phi}{\phi_0} = \frac{\oint_s \vec{E}\cdot d\vec{s}}{\oint_s \vec{E}_0\cdot d\vec{s}} = \frac{\oint_s \frac{\vec{E}_0}{\varepsilon_r}\cdot d\vec{s}}{\oint_s \vec{E}_0\cdot d\vec{s}} = \frac{1}{\varepsilon_r} \Rightarrow \boxed{\phi = \frac{\phi_0}{\varepsilon_r}}$$

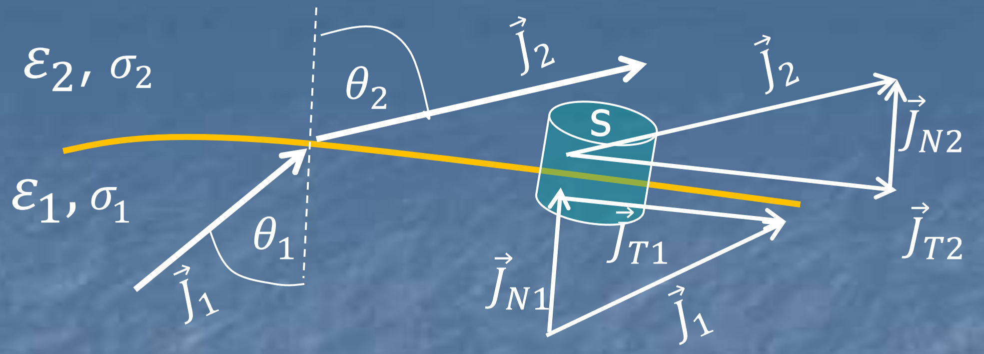
As for the effective charge q^*:

$$\frac{q^*}{\varepsilon_0} = \phi = \frac{\phi_0}{\varepsilon_r} = \frac{q/\varepsilon_0}{\varepsilon_r} = \frac{q}{\varepsilon_0\varepsilon_r} = \frac{q}{\varepsilon} \Rightarrow \boxed{q^* = \frac{\varepsilon_0 q}{\varepsilon} = \frac{q}{\varepsilon_r}}$$

And the capacitance:

$$C = \frac{Q}{V} = \frac{Q}{V_0/\varepsilon_r} = \boxed{\varepsilon_r C_0 = C}$$

Continuity of the current density $\vec{J}$



What happens to $\bar{J}$ in the interface conductor-insulator $(\sigma_2 \to 0)$?

From Kirchhoff Junction law we get:

$$0 = \oiint_{Cylinder} \vec{J} \cdot d\vec{s} = \oiint_{Cy} J_N ds = \int_{top} J_{N1} ds - \int_{bottom} J_{N2} ds \Rightarrow (J_{N1} - J_{N2}) \Rightarrow J_{N1} = J_{N2}$$

As for $\vec{J}_T$? On that we learn from the continuity of $\vec{E}_T$

$$E_{T1} = E_{T2} \Rightarrow \frac{J_{T1}}{\sigma_1} = \frac{J_{T2}}{\sigma_2}$$

In insulator $J = 0 \Longrightarrow J_{N1} = 0 = J_{N2}$. On the other hand $E_{N2} \neq 0, yet\ contradicts\ the\ fact\ that\ J_{N2} = 0$ since $J_{N2} = \sigma_2 E_{N2}$ but σ=0. In any case, at any point within a conductor $E = D = 0, i.e. E_1 = D_1 = 0,$ out of which $E_{T2} = 0\ and\ D_{N2} = 0$

The relation between the direction of J1 and J2 at the interface:

$$\tan\theta_1 = \frac{J_{T1}}{J_{N1}}; \quad \tan\theta_2 = \frac{J_{T2}}{J_{N2}} = \frac{J_{T1}\frac{\sigma_2}{\sigma_1}}{J_{N1}} = \tan\theta_1 \frac{\sigma_2}{\sigma_1} \Rightarrow$$

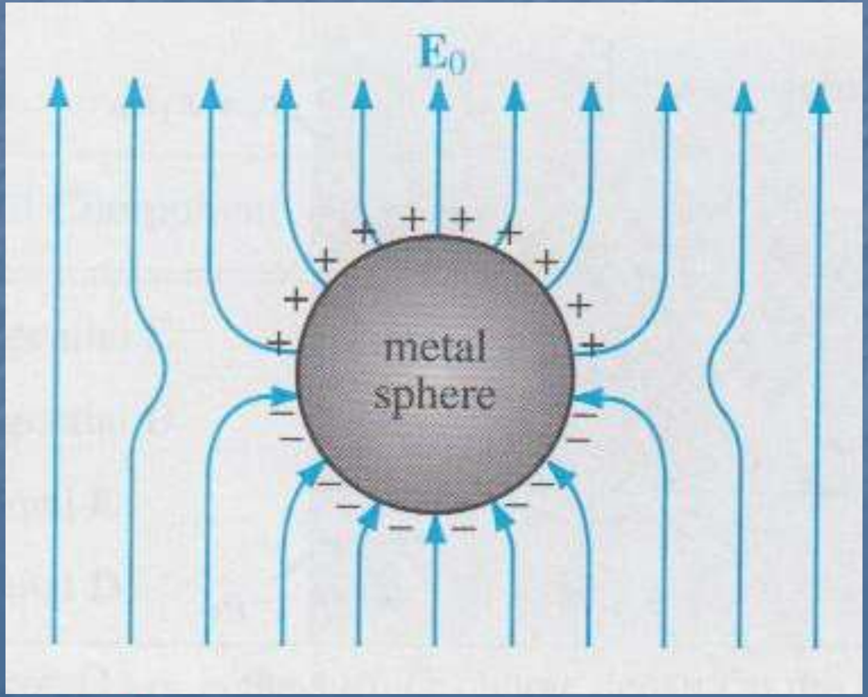
$$\tan\theta_2 = \tan\theta_1 \frac{\sigma_2}{\sigma_1}$$

Field Component	Any Two Media	Medium 1 Dielectric ε_1	Medium 2 Dielectric ε_2	Medium 1 Dielectric ε_1	Medium 2 Conductor
Tangential **E**	$E_{1t} = E_{2t}$	$E_{1t} = E_{2t}$		$E_{1t} = E_{2t} = 0$	
Tangential **D**	$D_{1t}/\varepsilon_1 = D_{2t}/\varepsilon_2$	$D_{1t}/\varepsilon_1 = D_{2t}/\varepsilon_2$		$D_{1t} = D_{2t} = 0$	
Normal **E**	$\hat{\mathbf{n}} \cdot (\varepsilon_1 \mathbf{E}_1 - \varepsilon_2 \mathbf{E}_2) = \rho_s$	$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$		$E_{1n} = \rho_s/\varepsilon_1$	$E_{2n} = 0$
Normal **D**	$\hat{\mathbf{n}} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$		$D_{1n} = \rho_s$	$D_{2n} = 0$

$$\tan\theta_1 = \frac{J_{T1}}{J_{N1}}; \quad \tan\theta_2 = \frac{J_{T2}}{J_{N2}} = \frac{J_{T1}\frac{\sigma_2}{\sigma_1}}{J_{N1}} = \tan\theta_1 \frac{\sigma_2}{\sigma_1} \Rightarrow$$

$$\tan\theta_2 = \tan\theta_1 \frac{\sigma_2}{\sigma_1}$$

Therefore, for $\sigma_1 \gg \sigma_2 \implies \theta_2 \rightarrow \frac{\pi}{2}$ and hence for every current arriving the conducting-isolator, it apparently flow along the interface. However, though current can not exist in insulator, electric field does.



In summary, the only component of the electric field exist is perpendicular to the interface conductor-isolator (see figure)

Does this result correlate with what we know about currents and equi-potetial surfaces?

The continuity of J in the interface between two conductors:

From the continuity of D ($\Delta D_N = \rho_s$):

$$\varepsilon_1 E_{1N} - \varepsilon_2 E_{2N} = \rho_s \ ; \text{int}\,roducing \ \ E = \frac{J}{\sigma}$$

$$\varepsilon_1 \frac{J_{1N}}{\sigma_1} - \varepsilon_2 \frac{J_{2N}}{\sigma_2} = \rho_s \ ; \text{sin}\,ce \ \ J_{1N} = J_{2N} = J_N$$

$$\rho_s = J_N \left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2} \right) = J_N \left(\frac{1}{\tau_1} - \frac{1}{\tau_2} \right)$$

In good conductors $\varepsilon \rightarrow \varepsilon_0$ *and hence*:

$$\rho_s = \varepsilon_0 J_N \left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2} \right)$$

Why a voltage difference is developed between the to sides of a resistor?

The duality between J and D (electrostatics):

Conductor	Insulator
$\oint \vec{E} \cdot d\vec{l} = 0$	$\oint \vec{E} \cdot d\vec{l} = 0$
J=σE=-σ gradV	D=εE=-ε gradV
div J=0	Div D=0
delta J_N =0	delta D_N=0
$J_{T1}/\sigma_1 = J_{T2}/\sigma_2$	$D_{1T}/\varepsilon_1 = D_{T2}/\epsilon_2$
$R = l/\sigma A$	$C = \epsilon A/l$
$RC = \frac{\epsilon}{\sigma} = \tau$	

Magnetism: Units

$$MKS:$$

$$\{\phi_B\} = Weber\,(Wb)$$

$$\{\vec{B}\} = \frac{\phi_B}{m^2} = \frac{Wb}{m^2} \equiv 1T\,(Tesla)$$

$$1T = 10^4\,Gauss(cgs)$$

$$\left|\vec{B}_{Earth}\right| \approx 0.5G$$

$$\{Wb\} = \frac{Kg \cdot m^2}{sec^2 \cdot A} = Volt \cdot sec = T \cdot m^2 = \frac{J}{A} = 10^8 Mx$$

Biot–Savart law and the vector potential $\bar{A}$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint_l \frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\oint_l d\vec{l}\times\left[-grad\left(\frac{1}{r}\right)\right] \quad [1]$$

$$From\ vector\ analysis: curl\left(\phi\vec{A}\right) = \phi curl\,\vec{A} - \vec{A}\times grad\,\phi \Rightarrow \vec{A}\times grad\,\phi = \phi curl\,\vec{A} - curl\left(\phi\vec{A}\right) \quad [2]$$

$$\vec{B} = -\frac{\mu_0 I}{4\pi}\oint_l\left[\frac{1}{r}\underbrace{curl\,d\vec{l}}_{=0} - curl\left(\frac{d\vec{l}}{r}\right)\right] = \oint_l \frac{\mu_0 I}{4\pi} curl\left(\frac{d\vec{l}}{r}\right) =$$

$$curl\oint_l \frac{\mu_0 I}{4\pi}\left(\frac{d\vec{l}}{r}\right) \equiv curl\vec{A}\ ;\ \boxed{\vec{B} = curl\vec{A}} \quad [3]$$

$$\vec{A} \equiv \oint_{l} \frac{\mu_0 I}{4\pi}\left(\frac{d\vec{l}}{r}\right) = \oiint_{l,s} \frac{\mu_0 J ds d\vec{l}}{4\pi r} = \oiint_{l,s} \frac{\mu_0 J \hat{l} ds dl}{4\pi r} = \iiint_{V} \frac{\mu_0 \vec{J} dv}{4\pi r}$$

$$V = \frac{Q}{4\pi\varepsilon r} = \iiint_{V} \frac{\rho_V dv}{4\pi\varepsilon r}$$

consequently one get the similarity between V and $\bar{A}$:

$$\nabla^2 V = -\frac{\rho_V}{\varepsilon} \Rightarrow \nabla^2 \vec{A} = -\mu \vec{J} \Rightarrow$$

$$\nabla^2 A_x = -\mu J_x \;;\; \nabla^2 A_y = -\mu J_y \;;\; \nabla^2 A_z = -\mu J_z$$

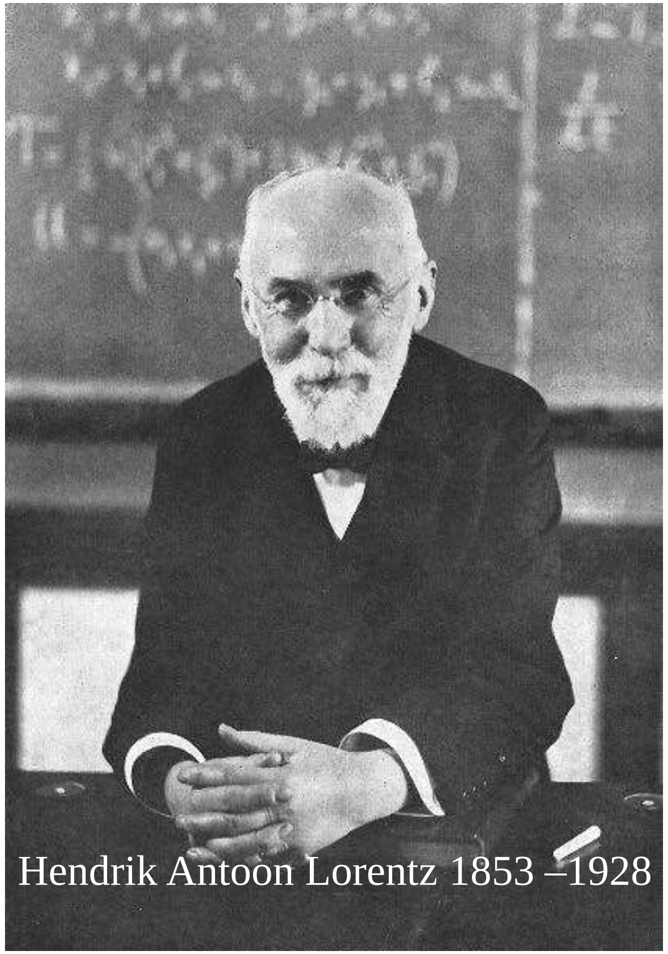
The integral and differential expression of Ampere's law:

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_f$$

Stokes theorem

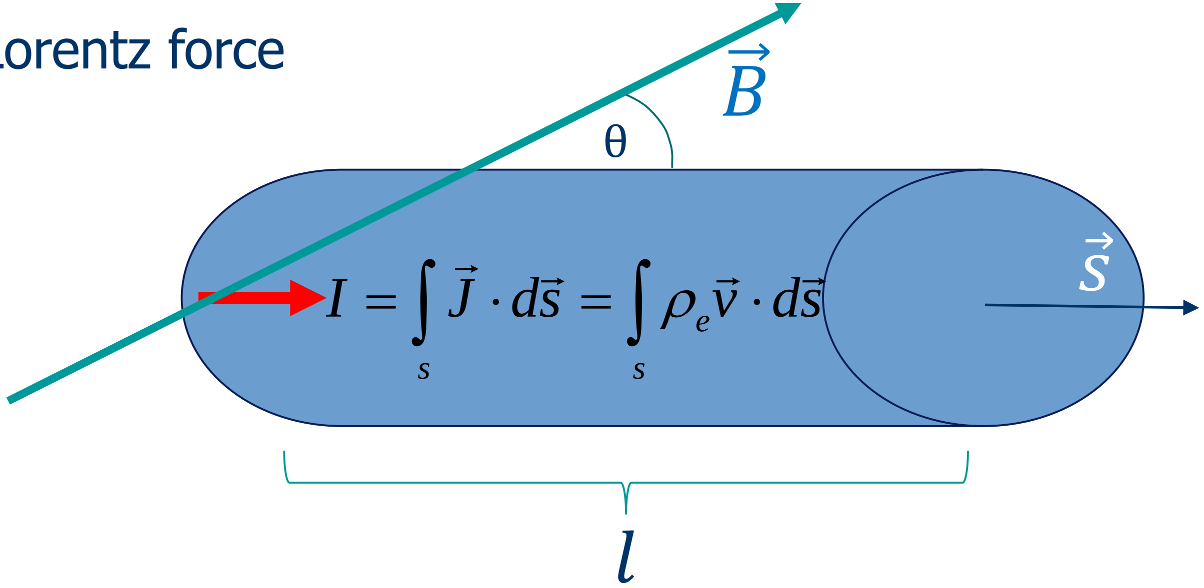
$$\iint_s (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \iint_s \vec{J} \cdot d\vec{s} \Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J}$$

Current is the source of magnetic field and charge is the source of electric field



Hendrik Antoon Lorentz 1853 –1928

Lorentz force



The magnetic force acting upon a current caring wire is given by:

$$\left|\vec{F}_B\right| = \left|I\vec{l} \times \vec{B}\right| = IlB\sin\theta = JSlB\sin\theta = \rho_e vVB\sin\theta = QvB\sin\theta = \left|Q\vec{v} \times \vec{B}\right|$$

And in the presence of eclectic field the total force is:

$$\bar{F} = Q(\bar{E} + \bar{v} \times \bar{B})$$

Faraday law

Michael Faraday (1791 – 1867)

$$V_{emf} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s}$$

Hence the circulation integral should include the contribution of the magnetic field as well:

Conservative field

$$V_{Total} = \oint_l \vec{E}_T \cdot d\vec{l} = 0 + V_{emf} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s} \Rightarrow \oint_l \vec{E}_T \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s}$$

Stokes theorem

$$= \iint_s \left(\nabla \times \vec{E}\right)\cdot d\vec{s} = -\frac{d}{dt}\iint_s \vec{B}\cdot d\vec{s} \Rightarrow \quad \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

Maxwell Equations

James Clerk Maxwell (1831–1879)

$$I : \oint_s \vec{E}\cdot d\vec{s} = \frac{q_{f,n}}{\varepsilon_0} = \phi_e \quad ; \quad div\vec{E} = \frac{\rho_V}{\varepsilon_0} \; ; \; [\oint_s \vec{D}\cdot d\vec{s} = q_{f,n} \; ; \; div\vec{D} = \rho_V] \; ; \; \vec{E} = -gradV \, ; \nabla^2 V = -\frac{\rho_V}{\varepsilon_0}$$

$$II : \oint_s \vec{B}\cdot d\vec{s} = 0 \quad ; \quad div\vec{B} = 0 \; ; \; [\oint_s \vec{H}\cdot d\vec{s} = 0 \, ; \, div\vec{H} = 0] \; ; \; \vec{B} = curl\vec{A} \, ; \nabla^2\vec{A} = -\mu_0\vec{J}$$

$$III : \oint_l \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int_s \vec{B}\cdot ds \quad ; \quad \nabla\times\vec{E} = -\frac{d\vec{B}}{dt} = -\frac{d\left(\nabla\times\vec{A}\right)}{dt} = -\nabla\times\dot{\vec{A}} \;\Rightarrow\; \vec{E}_{General} = -gradV - \dot{\vec{A}}$$

$$IV : \oint_l \vec{B}\cdot d\vec{l} = \mu_0 I + ? \quad ; \quad \nabla\times\vec{B} = \mu_0\vec{J} + ? \quad \rightarrow$$

$$\rightarrow \quad [\vec{H} \equiv \frac{\vec{B}}{\mu_0} \Rightarrow \oint_l \vec{H}\cdot d\vec{l} = I + ? \quad ; \quad \nabla\times\vec{H} = \vec{J} + ?]$$

Relating to the differential aspect of equation VI

$$curl\vec{H} = \vec{J} \rightarrow \underbrace{divcurl\vec{H}}_{=0} = \underbrace{div\vec{J}}_{\neq 0}$$

Following the continuity law

$$div\vec{J} + \dot{\rho}_V = 0 \rightarrow \frac{\rho_V}{\varepsilon_0} = div\vec{E} \rightarrow \dot{\rho}_V = \varepsilon_0 div\dot{\vec{E}} \Rightarrow$$

$$div(\vec{J} + \varepsilon_0\dot{\vec{E}}) = 0 \quad \Rightarrow \quad div(\vec{J} + \dot{\vec{D}}) = div(\vec{J}_C + \vec{J}_D) = 0$$

$$I_D = \frac{d}{dt}\int_s \vec{D}\cdot d\vec{s} = \varepsilon_0 \frac{d}{dt}\int_s \vec{E}\cdot d\vec{s} = \varepsilon_0 \frac{d\phi_e}{dt}$$

$$IV: \oint_l \vec{B}\cdot d\vec{l} = \mu_0(I_C + I_D) = \mu_0(I_C + \varepsilon_0\frac{d\phi_e}{dt}) = \mu_0(I_C + \varepsilon_0\frac{d}{dt}\int_s \vec{E}\cdot d\vec{s}) \Rightarrow$$

$$\Rightarrow \quad \nabla\times\vec{B} = \mu_0(\vec{J}_C + \vec{J}_D) = \mu_0(\vec{J}_C + \varepsilon_0\dot{\vec{E}}) = \mu_0(\vec{J}_C + \dot{\vec{D}})$$

The magnetic field intensity-H

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 I_f \Rightarrow \oint_L \frac{\vec{B}}{\mu_0}\cdot d\vec{l} = I_f \; ; \quad \boxed{\frac{\vec{B}}{\mu_0} \equiv \vec{H} \Rightarrow \vec{B} = \mu_0 \vec{H}}$$

$$\Rightarrow \boxed{\oint_L \vec{H}\cdot d\vec{l} = I_f} \Rightarrow \{H\} = \frac{A}{m}$$

$$\boxed{\oint_L \vec{B}\cdot d\vec{l} = \mu_0 I_f} \rightarrow Via\ Stcokes\ Theorm: \oint_L \vec{B}\cdot d\vec{l} = \iint_{S(L)} \nabla X\vec{B}\cdot d\vec{s} = \iint_{S(L)} \mu_0 \vec{J}_f \cdot d\vec{s} \Rightarrow$$

$$\Rightarrow \boxed{\nabla X\vec{B} = \mu_0 \vec{J}_f}$$

$$\boxed{\nabla X\vec{H} = \vec{J}_f}$$

B : *The magnetic flux density*

H : *The magnetic field intensity*

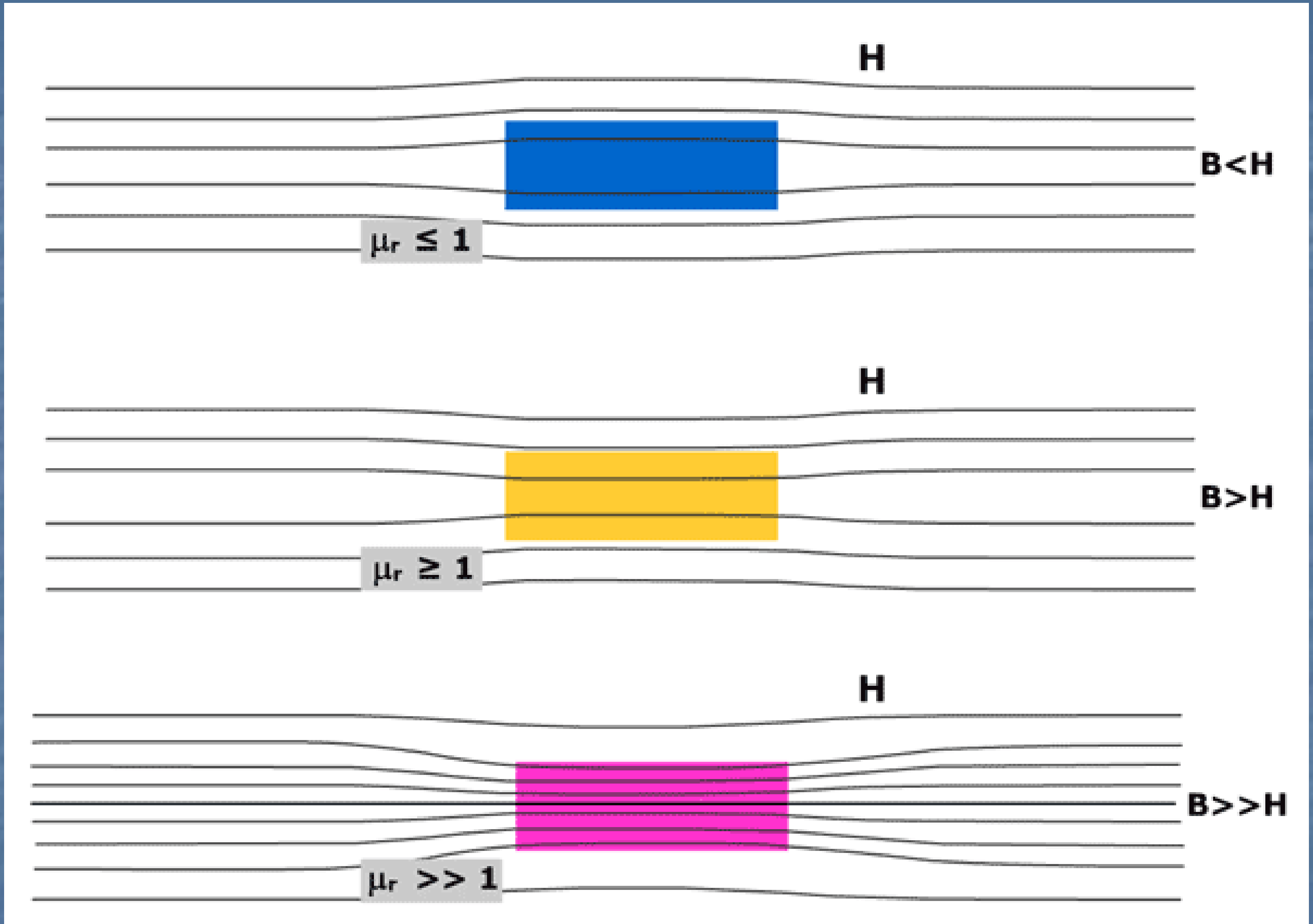
Types of magnetism:

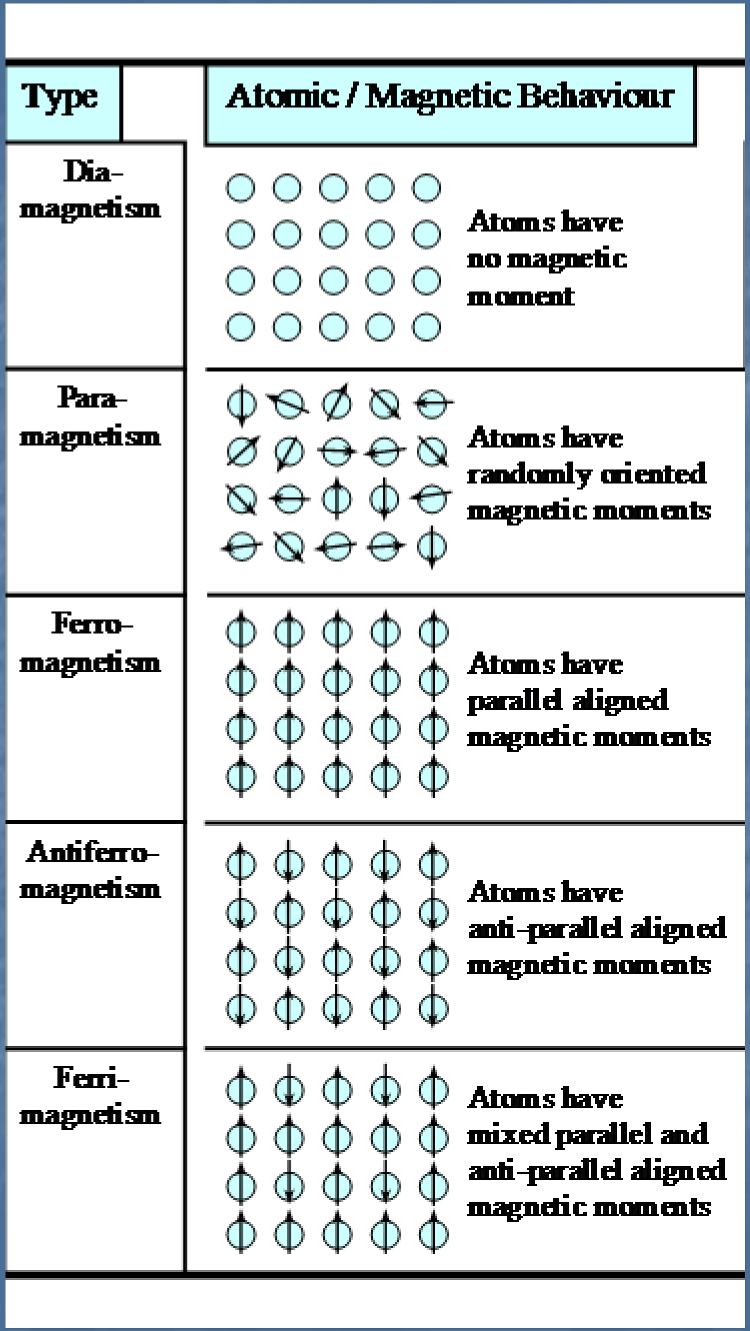
Diamagnetism	Paramagnetism	Ferromagnetism	
No	Yes but weak	Yes and constant	Magnetic field
Electrons spin orbital	Electrons spin	Magnetizing regions	Source of magnetism
Opposite	Identical	Varies, Hysteresis	Direction of B in respect to H
Cupper, Lead, Diamond, Mercury, Silicon	Aluminum, Calcium, Magnesium, Tungsten	Iron, Nickel, Cobalt	Typical materials
$\approx -10^{-5}$	$\approx 10^{-5}$	$\lvert \chi_m \rvert \gg 1$	Typical χ_m
≈ 1	≈ 1	$\lvert \mu_r \rvert \gg 1$	Typical μ_r

Diamagnetism

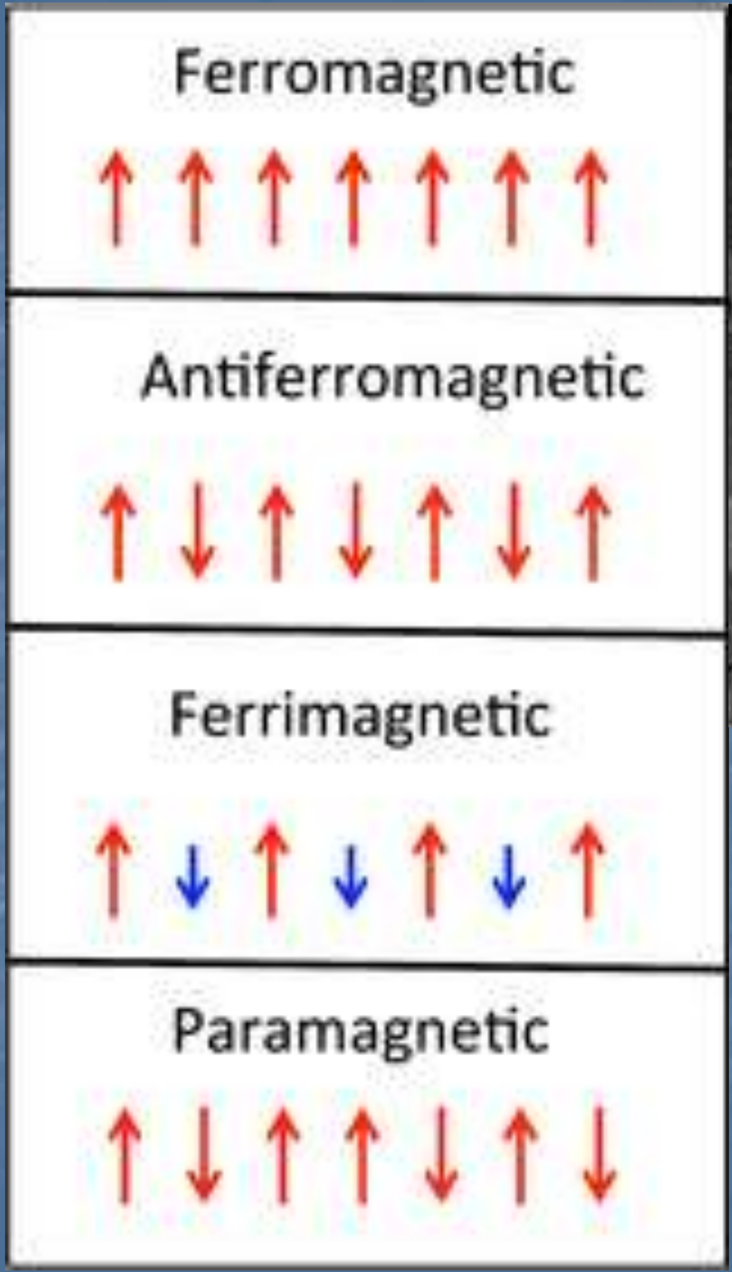
Paramagnetism

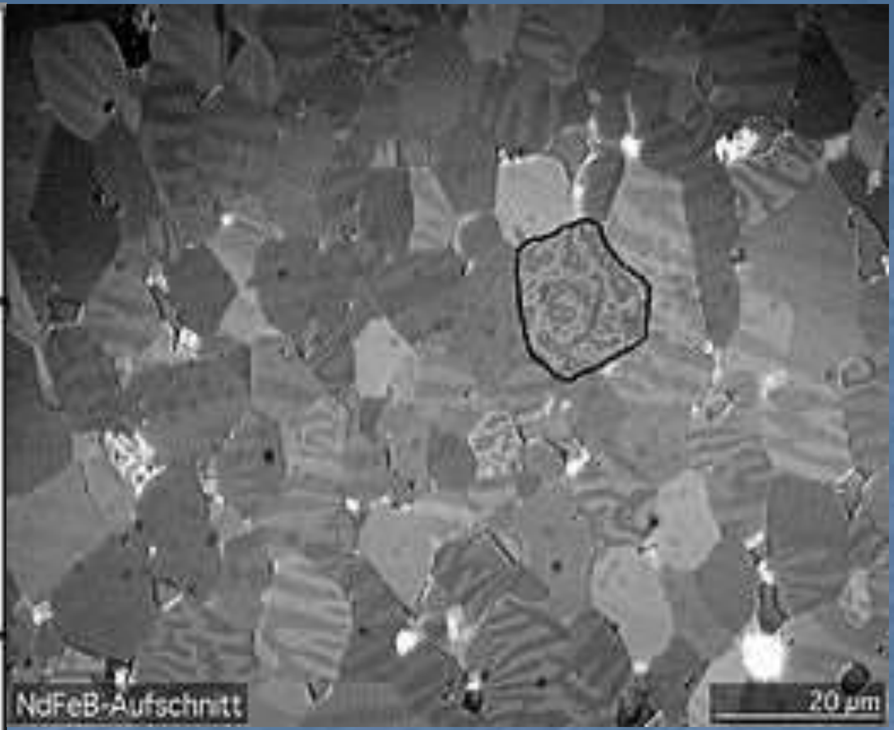
Ferromagnetism





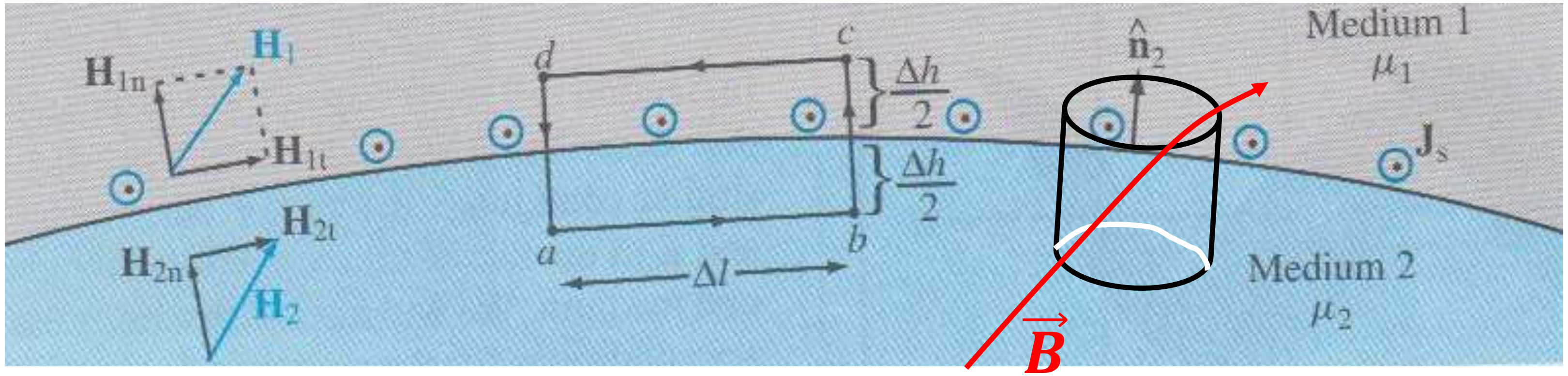
Type	Atomic / Magnetic Behaviour
Dia-magnetism	Atoms have no magnetic moment
Para-magnetism	Atoms have randomly oriented magnetic moments
Ferro-magnetism	Atoms have parallel aligned magnetic moments
Antiferro-magnetism	Atoms have anti-parallel aligned magnetic moments
Ferri-magnetism	Atoms have mixed parallel and anti-parallel aligned magnetic moments





Within each grain are a series of lighter and darker stripes (imaged by using the optical Kerr effect) that are ferromagnetic domains with opposite orientations. Averaged over the whole sample, these domains have random orientation so the net magnetization is zero. Moving domain exist as well.

Boundary conditions for B and H



$$0 = \oint_S \vec{B} \cdot d\vec{s} \Rightarrow \boxed{B_{N1} = B_{N2}} \Rightarrow \boxed{\mu_1 H_{N1} = \mu_2 H_{N2}}$$

$$\oint_L \vec{H} \cdot d\vec{l} = I_f \Rightarrow (H_{T1} - H_{T2})L = I_f \Rightarrow (H_{T1} - H_{T2}) = \frac{I_f}{L} = i_f \Rightarrow \boxed{\Delta H_T = i_f} \Rightarrow$$

$$\Rightarrow \boxed{\frac{B_{T1}}{\mu_1} - \frac{B_{T2}}{\mu_2} = i_f}$$

$$For\; i_f = 0: \quad \boxed{\Delta H_T = 0} \quad and \quad \boxed{\frac{B_{T1}}{\mu_1} = \frac{B_{T2}}{\mu_2}}$$